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	<i>Theoretical.</i>	<i>Observed.</i>
Neptune, mean,	6,450.776	6,453.751
Saturn, mean,	2,053.346	2,049.514
Asteroid, mean,	653.600	654.760
Earth, perihelion,	208.048	207.583
Mercury, perihelion,	66.224	68.483
Sun, major axis,	2.136	2.132
Primary center, osc.,	.679	.679

The slight discrepancies in the values of T'' , V' , V'' , seem to be attributable to Jupiter's mean eccentricity, but they are all within the limits of uncertainty of observation.

PERFECT CUBES.

BY PROF. W. D. HENKLE, SALEM, OHIO.

The following interesting facts in reference to perfect cubes were discovered by the writer about seven years ago:

Perfect cubes ending in 1, 2, 3, 4, 5, 6, 7, 8, 9 or 0 followed by 1
have roots ending in 7, 4, 1, 8, 5, 2, 9, 6, 3 or 0 followed by 1.

Perfect cubes ending in 1, 2, 3, 4, 5, 6, 7, 8, 9 or 0 followed by 3
have roots ending in 1, 4, 7, 0, 3, 6, 9, 2, 5 or 8 followed by 7.

Perfect cubes ending in 1, 2, 3, 4, 5, 6, 7, 8, 9 or 0 followed by 7
have roots ending in 7, 0, 3, 6, 9, 2, 5, 8, 1 or 4 followed by 3.

Perfect cubes ending in 1, 2, 3, 4, 5, 6, 7, 8, 9 or 0 followed by 9
have roots ending in 3, 0, 7, 4, 1, 8, 5, 2, 9 or 6 followed by 9.

Hence every perfect cube ending in 11 has 71 as the ending of its root; every perfect cube ending in 21 has 41 as the ending of its root; and so on. Thus we see that the last two figures of any perfect cube ending in 1, 3, 7 or 9 may be known if the figure before 1, 3, 7 or 9 is also known. For instance the cube root of the perfect cube 185193 may be known to be 57 because all perfect cubes ending in 93 have roots ending in 57. Thus we may know the cube root of the perfect cube 3,869,893 by knowing only 3, ... 93. The first figure of the root must be 1 and the next two 57.

If the series 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 be repeated or made circular it

may be seen that the series 7, 4, 1, 8, 5, 2, 9, 6, 3, 0 may be obtained by beginning at 7 in the circular series and counting forward successively to the 7th following figure; in the same way the series 1, 4, 7, 0, 3, 6, 9, 2, 5, 8 can be found by counting forward by threes beginning at 1. To get the series 7, 0, 3, 6, 9, 2, 5, 8, 1, 4 begin with 7 and count by threes. To get the series 3, 0, 7, 4, 1, 8, 5, 2, 9, 6 begin at 3 and count by sevens.

It is desirable to obtain some method of getting directly the ten's figure of the cube. The following scheme accomplishes this:

1	2	3	4	5	6	7	8	9	0
7	7	7	7	7	7	7	7	7	7
<u>7</u>	<u>.4</u>	<u>.1</u>	<u>.8</u>	<u>.5</u>	<u>.2</u>	<u>.9</u>	<u>.6</u>	<u>.3</u>	<u>0</u>
1	2	3	4	5	6	7	8	9	0
3	3	3	3	3	3	3	3	3	3
<u>3</u>	<u>6</u>	<u>9</u>	<u>.2</u>	<u>.5</u>	<u>.8</u>	<u>.1</u>	<u>.4</u>	<u>.7</u>	<u>0</u>
8	8	8	8	8	8	8	8	8	8
<u>.1</u>	<u>.4</u>	<u>.7</u>	<u>.0</u>	<u>.3</u>	<u>.6</u>	<u>.9</u>	<u>.2</u>	<u>.5</u>	<u>8</u>
1	2	3	4	5	6	7	8	9	0
3	3	3	3	3	3	3	3	3	3
<u>3</u>	<u>6</u>	<u>9</u>	<u>.2</u>	<u>.5</u>	<u>.8</u>	<u>.1</u>	<u>.4</u>	<u>.7</u>	<u>0</u>
4	4	4	4	4	4	4	4	4	4
<u>7</u>	<u>.0</u>	<u>.3</u>	<u>.6</u>	<u>.9</u>	<u>.2</u>	<u>.5</u>	<u>.8</u>	<u>.1</u>	<u>4</u>
1	2	3	4	5	6	7	8	9	0
7	7	7	7	7	7	7	7	7	7
<u>7</u>	<u>.4</u>	<u>.1</u>	<u>.8</u>	<u>.5</u>	<u>.2</u>	<u>.9</u>	<u>.6</u>	<u>.3</u>	<u>0</u>
6	6	6	6	6	6	6	6	6	6
<u>.3</u>	<u>.0</u>	<u>.7</u>	<u>.4</u>	<u>.1</u>	<u>.8</u>	<u>.5</u>	<u>.2</u>	<u>.9</u>	<u>6</u>

In the first case multiply by 7 and reject the left-hand digit when there is one; in the second case multiply 3 and add 8; in the third multiply 3 and add 4; and in the fourth multiply 7 and add 6, and reject as before when necessary.

The process may be stated in another way: Putting D as the ten's figure of the cube, and sub-figures as the last figures of the cubes.

To find the ten's figure of the root of a cube ending in D_1 multiply D by 7 and cast out the tens.

When the cube ends in D_3 , multiply D by 3, add 8, and cast out the tens.

"	"	"	"	D_7 ,	"	"	3, add 4,	"	"	"
"	"	"	"	D_9 ,	"	"	7, add 6,	"	"	"

For instance to find the cube root of 912673: The last figure of the root must be 7; to get the ten's figure multiply 7 by 3, add 8, and cast out the tens or reject the left-hand digit; this gives 9, hence the root is 97.

Perfect cubes ending in 1, 3, 5, 7, or 9 followed by 2
have roots ending in $\left\{ \begin{array}{l} 5, 1, 7, 3, \text{ or } 9 \\ 0, 6, 2, 8, \text{ or } 4 \end{array} \right\}$ followed by 8.

Perfect cubes ending in 2, 4, 6, 8, or 0 followed by 4
have roots ending in $\left\{ \begin{array}{l} 7, 1, 5, 9, \text{ or } 3 \\ 2, 6, 0, 4, \text{ or } 8 \end{array} \right\}$ followed by 4.

Perfect cubes ending in 1, 3, 5, 7, or 9 followed by 6
have roots ending in $\left\{ \begin{array}{l} 5, 9, 3, 7, \text{ or } 1 \\ 0, 4, 8, 2, \text{ or } 6 \end{array} \right\}$ followed by 6.

Perfect cubes ending in 2, 4, 6, 8, or 0 followed by 8
have roots ending in $\left\{ \begin{array}{l} 6, 2, 8, 4, \text{ or } 0 \\ 1, 7, 3, 9, \text{ or } 5 \end{array} \right\}$ followed by 2.

In these cases there are alternative series. In the case of D_2 the corresponding number of the first of the alternative series can be obtained by multiplying D by 3, and adding 2, and rejecting the tens; in the case of D_4 and D_6 , by multiplying D by 2, adding 3, and rejecting the tens, and in the case of D_8 , by multiplying D by 3, and rejecting the tens.

To obtain the corresponding alternative of the second series subtract 5 when possible, if not add 5.

What is the cube root of the perfect cube 110592? Multiplying 9 by 3, adding 2, and rejecting the tens we have 9. The alternative is 5 less, or 4. Hence the root is either 98 or 48. The 110 shows that it must be 48. What is the cube root of the perfect cube 25,412,184? Multiplying 8 by 2, adding 3, and rejecting the tens we get 9. Hence the ten's figure of the root is either 9 or 4. From 25 we get the first figure 2, and from 4 the last figure 4. The root is either 294 or 254. We decide in favor of 294 because 25 is nearly the cube of 3. There is another mode of deciding which is the root: Assume either one as the cube root, subtract it from the cube and the remainder is divisible by 6 if the root assumed is right. A cube when divided by 6 gives the same remainder as when its root is divided by 6.

Perfect cubes ending in 2 or 7 followed by 5

have roots ending in $\left\{ \begin{array}{l} 0 \text{ or } 1 \\ 2 \text{ or } 3 \\ 4 \text{ or } 5 \\ 6 \text{ or } 7 \\ 8 \text{ or } 9 \end{array} \right\}$ followed by 5.

This is the most difficult case. When a perfect cube ends in 25 its root must end in an even number (0 being included) followed by 5. If a perfect cube ends in 75, its root must end in an odd number followed by 5. Find the cube root of 61,629,875. The root is 315, 335, 355, 375 or 395. Dividing 61,629,875 by 6 we get 5 for a remainder. The same remainder is obtained from 335 and 395. Hence we have only to decide between 335 and 395. The 61 plainly indicates 395.

In a future article I shall discuss the relation of the hundreds of the cube to the hundreds of the root.

LETTER FROM PROF. SCHIAPARELLI.

EDITOR OF THE ANALYST, DES MOINES, IOWA:—Prof. A Hall of Washington, has had the kindness to send me No. 2 of your journal, THE ANALYST, containing an article with the title "Comets and Meteors." In this article Mr. Hall offers some objections to the difficulties which I have opposed to the calculation of Laplace on the probabilities of hyperbolic orbits, (*Conn. des Temps* for 1816, pp. 215–218). I ask of you permission to add some observations in reply to the remarks of Mr. Hall.

The first remark is thus stated: "If S be the sun and P the point on the surface of the sphere of activity of the sun's attractive force, the bodies that pass through P may have all possible directions, but a direction making a small angle with PS is less probable than one making a greater angle. I cannot see that Prof. Schiaparelli has introduced this condition into his solution." I have always been persuaded of the truth of the proposition announced by Mr. Hall. If I have not explicitly spoken of it, it is only because the course of the demonstration does not require it. My solution would have been erroneous if I had supposed the contrary proposition; that is to say, that all the angles with PS are equally probable. Mr. Hall will have some difficulty to prove that I have committed an error of this gravity. In order to establish that a demonstration is in fault it is not sufficient to say that this or that idea is not indicated; it is necessary to find some evident error either in the fundamental supposition or in the logical connection of the reasoning.

The second remark of Mr. Hall refers to a detail of calculation; that is to say, to this, that in seeking to find the defect of the solution of Laplace I have not spoken of the division by U , indicated by this great geometer at the beginning of page 216 of his memoir. I shall observe here that U is the greatest value physically attributable to the velocity v ; it is there